Robotics

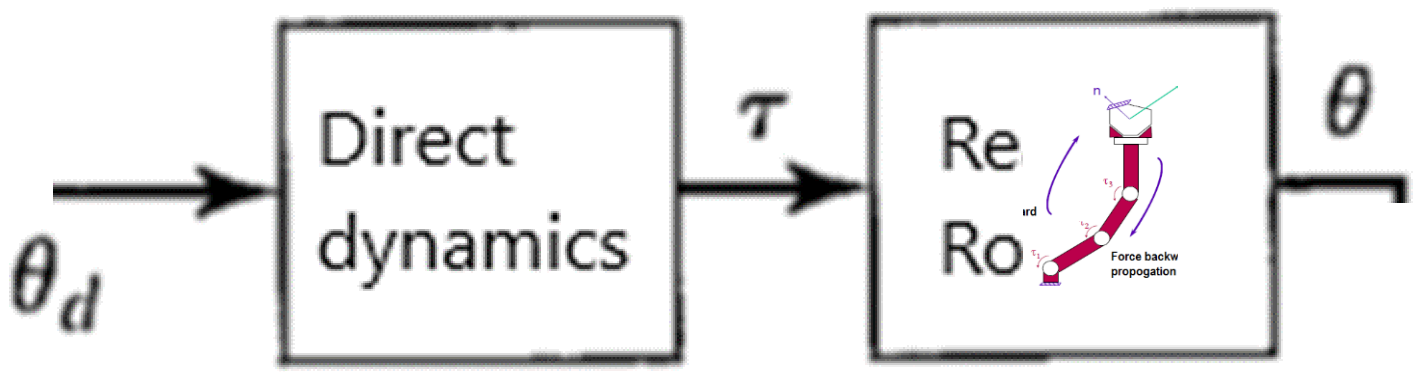
Chapter 6
Introduction to Manipulator dynamics and control

6.1 INTRODUCTION

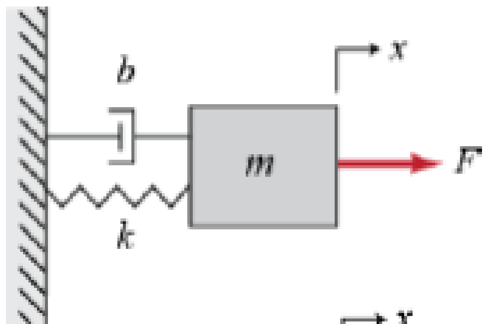
Our study of manipulators so far has focused on kinematic considerations only. We have studied static positions, static forces, and velocities; but we have never considered *the forces required to cause motion*. In this chapter, we consider the equations of motion for a manipulator—the way in which motion of the manipulator arises from torques applied by the actuators or from external forces applied to the manipulator.

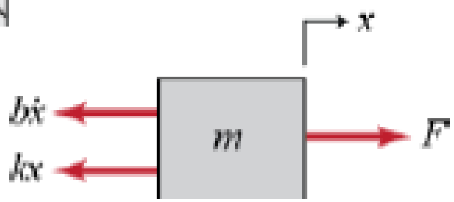
Direct dynamic model

There are two problems related to the dynamics of a manipulator that we wish to solve. In the first problem, we are given a trajectory point, Θ, $\dot{\Theta}$, and $\ddot{\Theta}$, and we wish to find the required vector of joint torques, τ. this is called Direct Dynamics



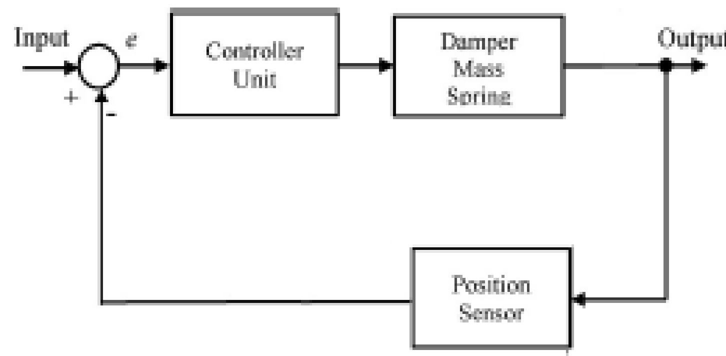
Mass-spring-damper dynamic model

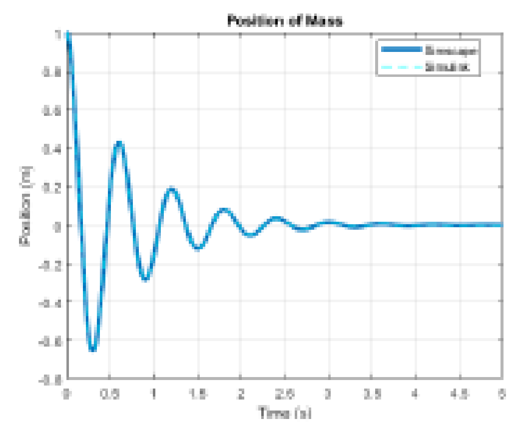




$$\Sigma F_x = F(t) - b\dot{x} - kx = m\ddot{x}$$

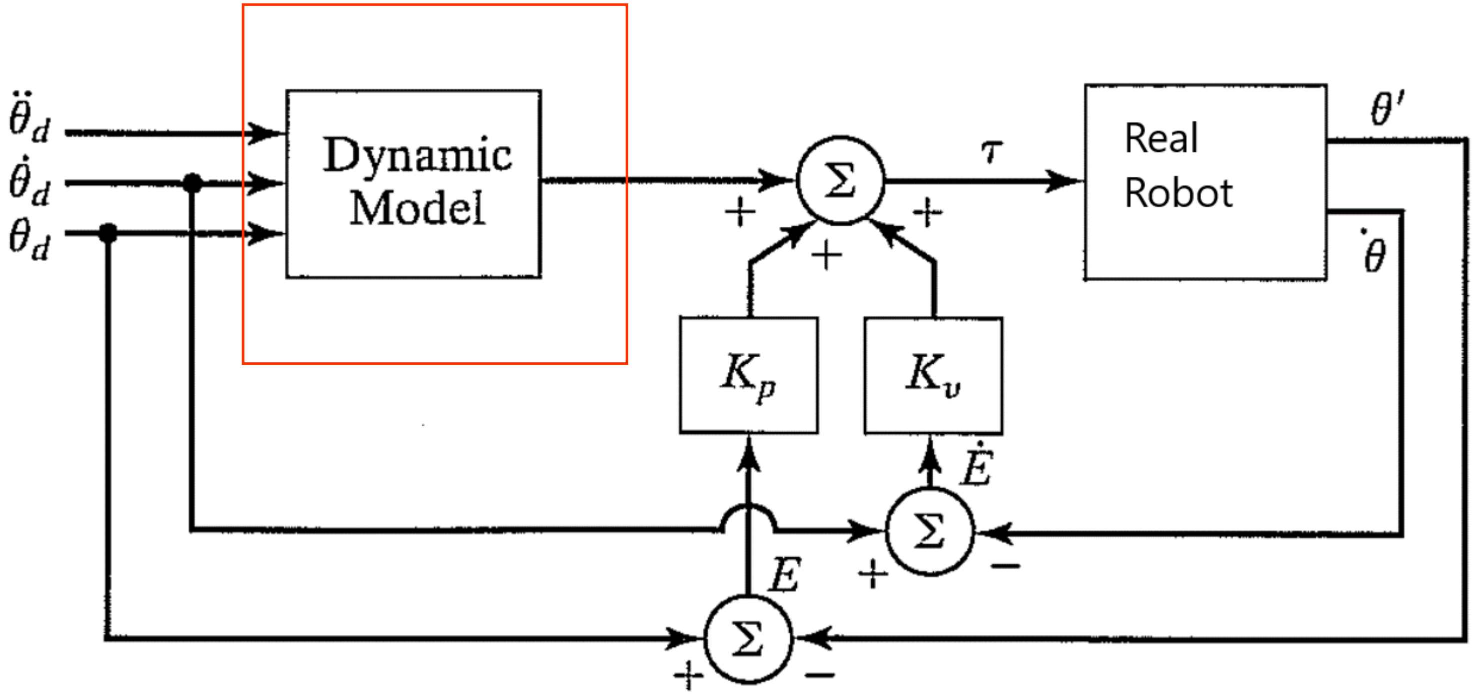
$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$





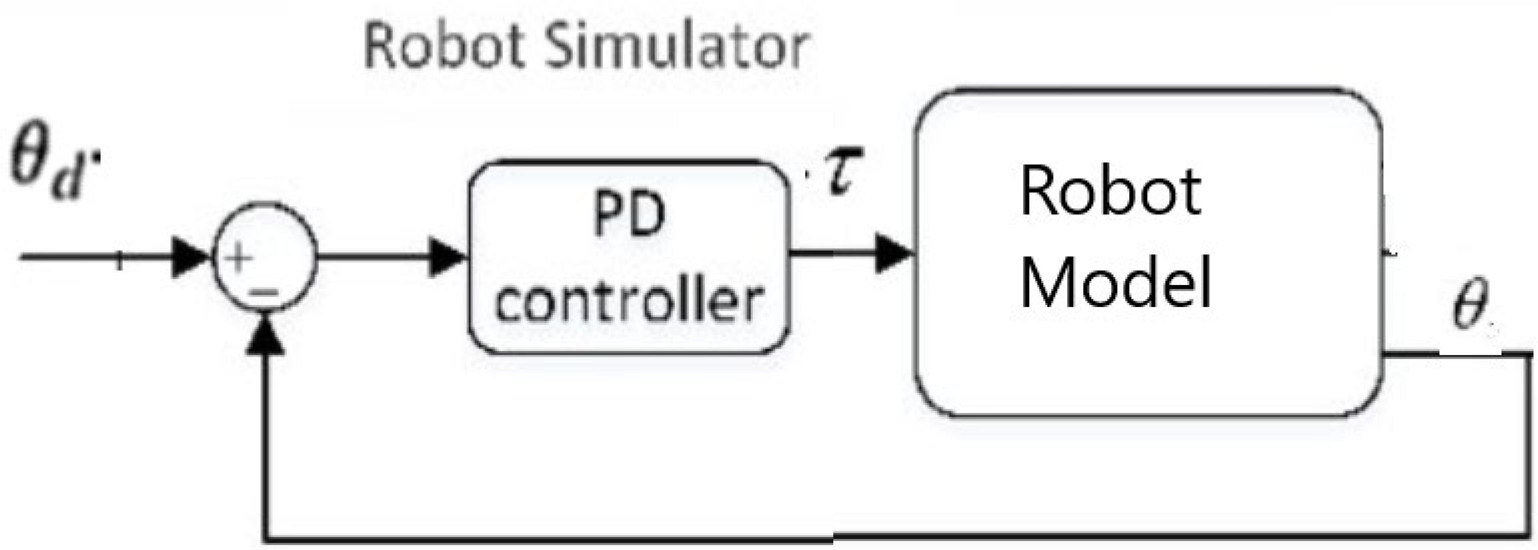
Direct dynamic model

This formulation of dynamics
is useful for the problem of controlling the manipulator

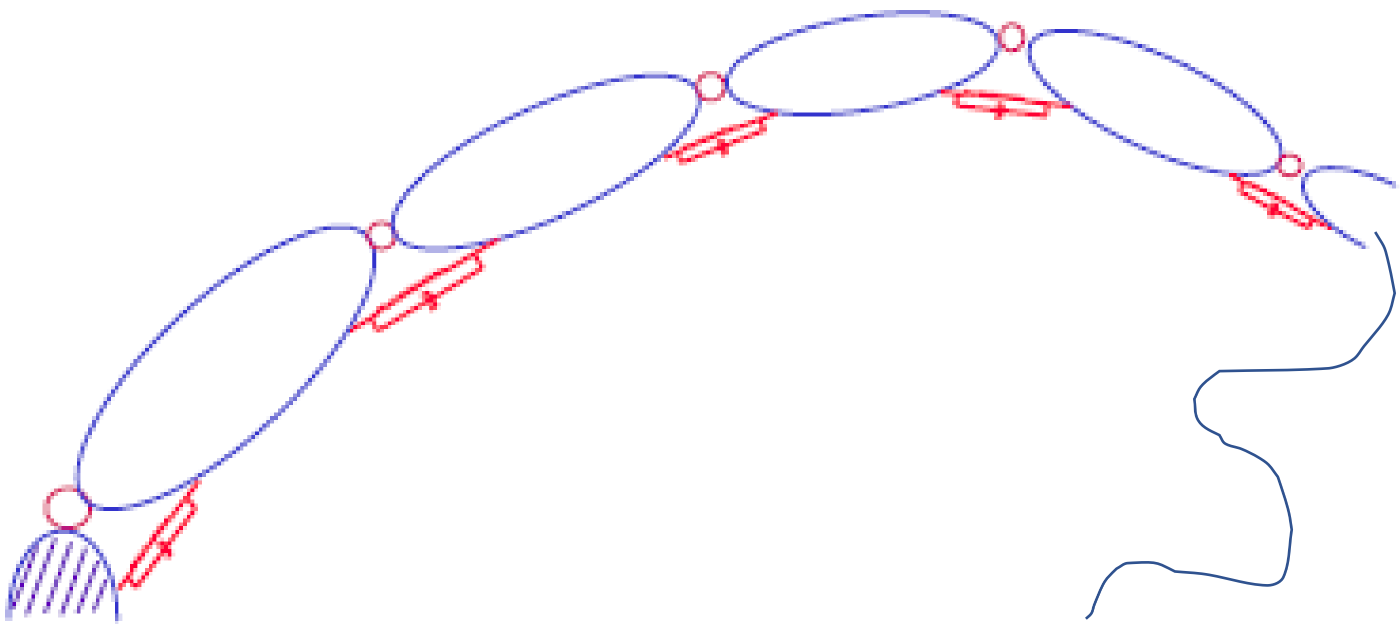


Inverse dynamic model

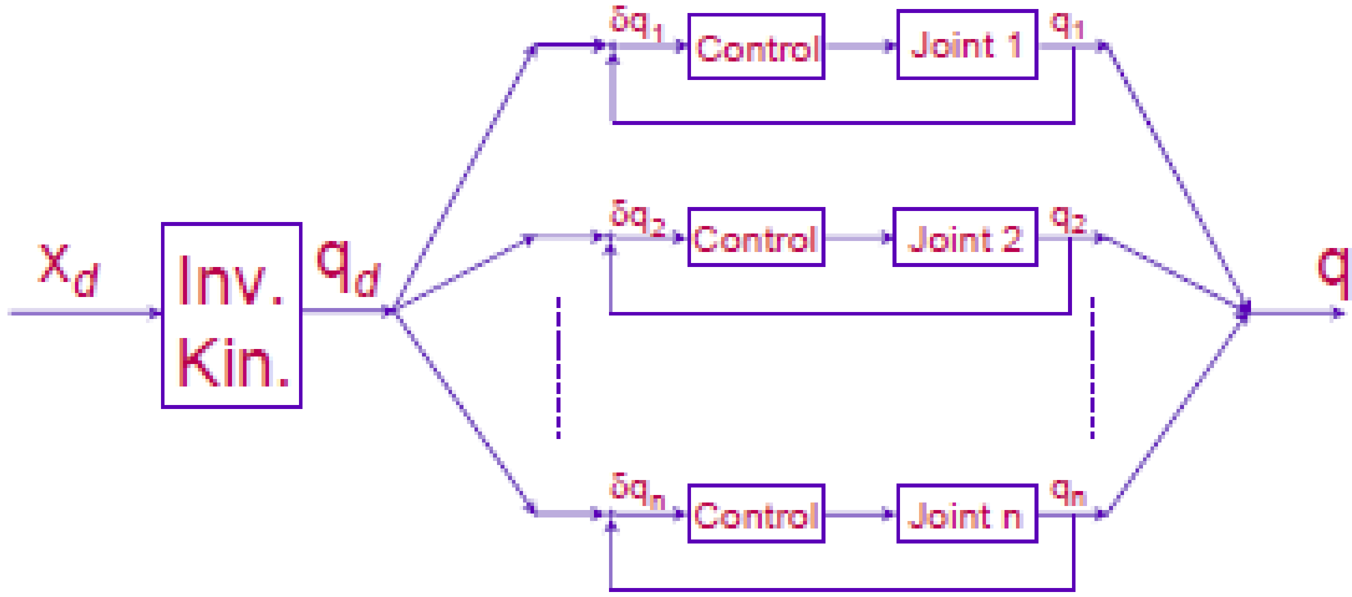
The second problem is to calculate how the mechanism will move under application of a set of joint torques. That is, given a torque vector, τ, calculate the resulting motion of the manipulator, Θ, $\dot{\Theta}$, and $\ddot{\Theta}$. This is useful for simulating the manipulator.

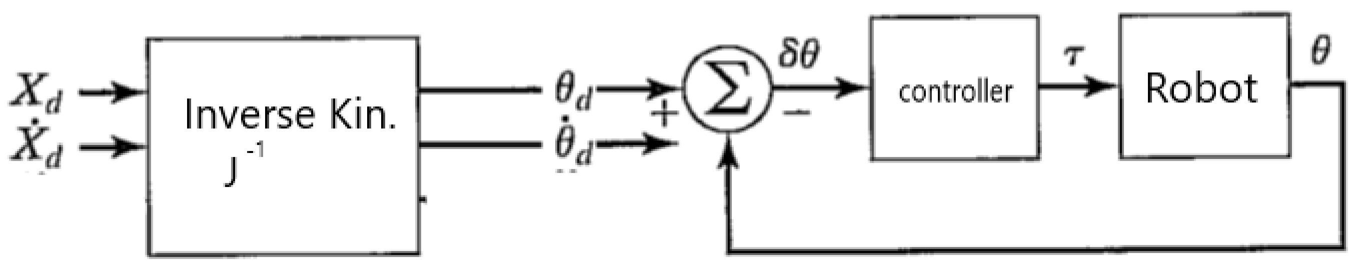


Joint-Space Control

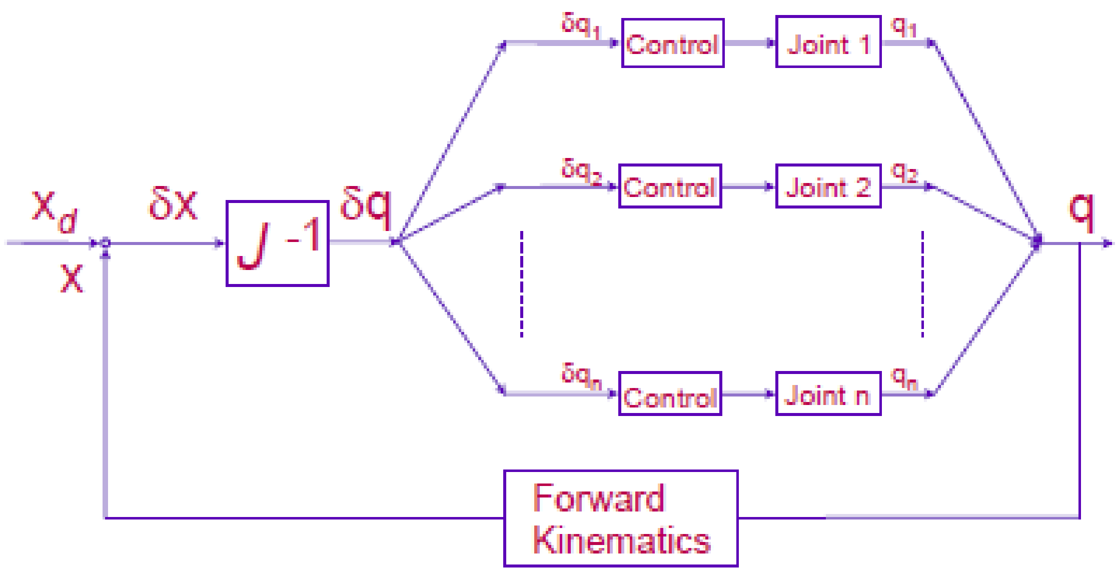


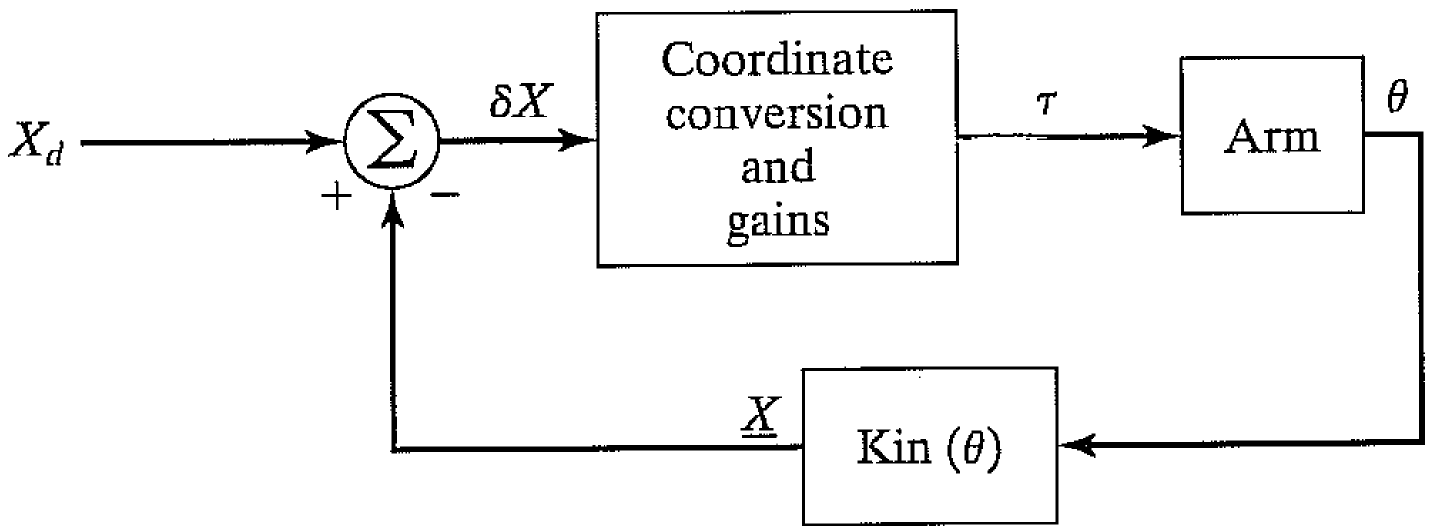
Joint Space Control





Cartesian space control

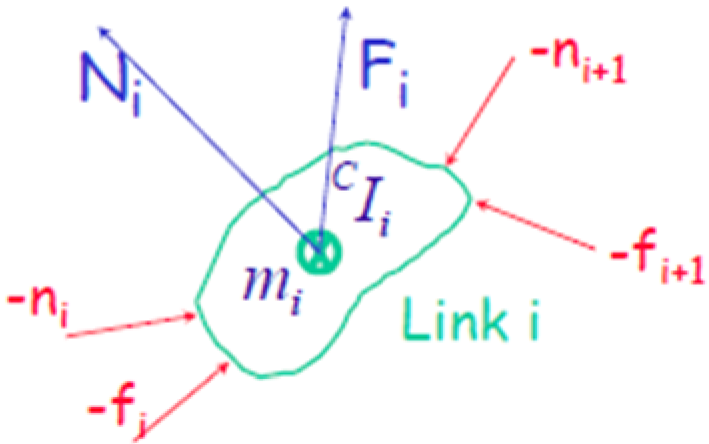




Two methods for Dynamics modeling

Formulations

Newton-Euler



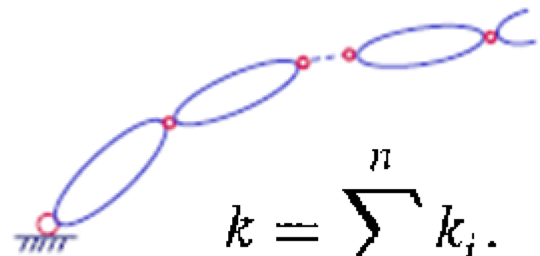
Newton: $F_i = m_i \dot{v}_{C_i}$

Euler: $N_i = {}^{C_i}I\dot{\omega}_i + \omega_i \times {}^{C_i}I\omega_i$

Eliminate Internal
Forces and Moments

$$\Gamma_i = \begin{cases} n_i^T . Z_i & \text{revolute} \\ f_i^T . Z_i & \text{prismatic} \end{cases}$$

Lagrange



$$k = \sum_{i=1}^{n} k_i .$$

$$u = \sum_{i=1}^{n} u_i .$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\Theta}} - \frac{\partial \mathcal{L}}{\partial \Theta} = \tau,$$

$$\mathcal{L}(\Theta, \dot{\Theta}) = k(\Theta, \dot{\Theta}) - u(\Theta).$$

Two methods for Dynamics modeling

$$ {}^{i}f_i = {}^{i}_{i+1}R\, {}^{i+1}f_{i+1} + {}^{i}F_i, $$

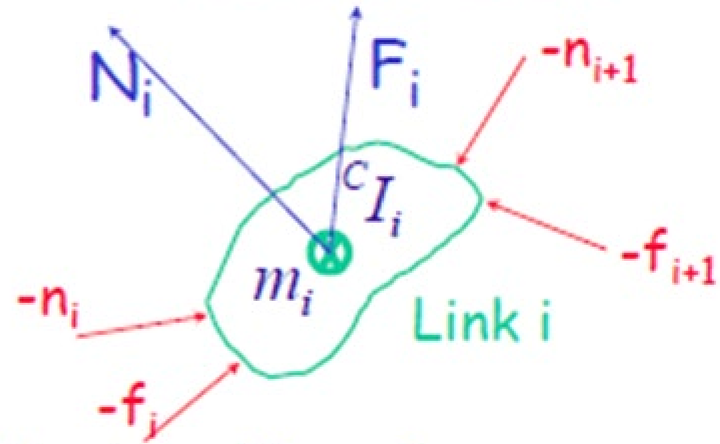
$$ {}^{i}n_i = {}^{i}N_i + {}^{i}_{i+1}R\, {}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i $$
$$ + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\, {}^{i+1}f_{i+1}, $$

$$ \tau_i = {}^{i}n_i^T\, {}^{i}\hat{Z}_i. $$

$$ \Gamma_i = \begin{cases} n_i^T . Z_i & \text{revolute} \\ f_i^T . Z_i & \text{prismatic} \end{cases} $$

Formulations

Newton-Euler

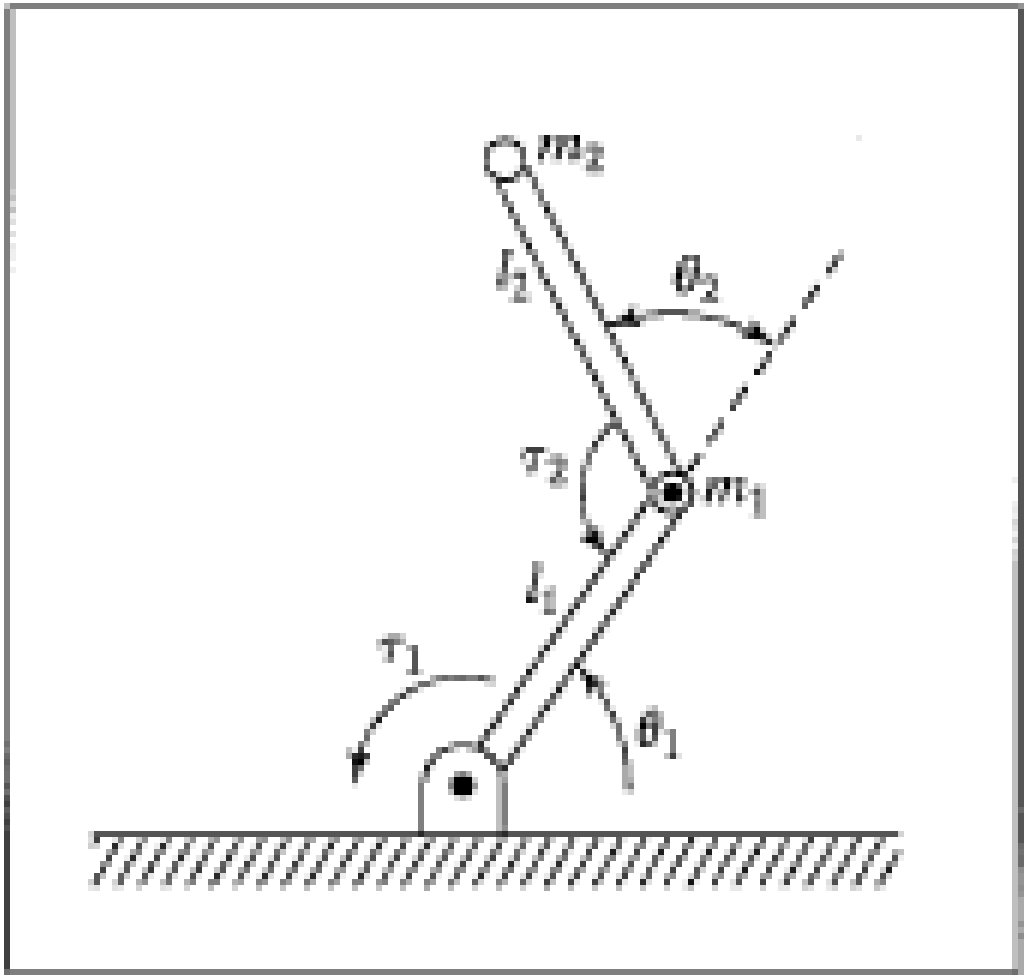


Newton: $F_i = m_i \dot{v}_{C_i}$

Euler: $N_i = {}^{C_i}I\dot{\omega}_i + \omega_i \times {}^{C_i}I\omega_i$

Example1

Here we compute the closed-form dynamic equations for the two-link planar manipulator shown in Fig. 6.6. For simplicity, we assume that the mass distribution is extremely simple: All mass exists as a point mass at the distal end of each link. These masses are m_1 and m_2.



Extracting the $\hat{Z}$ components of the ${}^i n_i$, we find the joint torques:

$$\tau_1 = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2(2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2$$
$$-2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1,$$
$$\tau_2 = m_2 l_1 l_2 c_2 \ddot{\theta}_1 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 l_2 g c_{12} + m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2). \quad (6.58)$$

Equations (6.58) give expressions for the torque at the actuators as a function of joint position, velocity, and acceleration. Note that these rather complex functions arose from one of the simplest manipulators imaginable. Obviously, the closed-form equations for a manipulator with six degrees of freedom will be quite complex.

THE STRUCTURE OF A MANIPULATOR'S DYNAMIC EQUATIONS

It is often convenient to express the dynamic equations of a manipulator in a single equation that hides some of the details, but shows some of the structure of the equations.

The state-space equation

When the Newton–Euler equations are evaluated symbolically for any manipulator, they yield a dynamic equation that can be written in the form

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta), \tag{6.59}$$

where $M(\Theta)$ is the $n \times n$ **mass matrix** of the manipulator, $V(\Theta, \dot{\Theta})$ is an $n \times 1$ vector of centrifugal and Coriolis terms, and $G(\Theta)$ is an $n \times 1$ vector of gravity terms. We use the term **state-space equation** because the term $V(\Theta, \dot{\Theta})$, appearing in (6.59), has both position and velocity dependence [3].

Each element of $M(\Theta)$ and $G(\Theta)$ is a complex function that depends on Θ, the position of all the joints of the manipulator. Each element of $V(\Theta, \dot{\Theta})$ is a complex function of both Θ and $\dot{\Theta}$.

We may separate the various types of terms appearing in the dynamic equations and form the mass matrix of the manipulator, the centrifugal and Coriolis vector, and the gravity vector.

$$M(\Theta) = \begin{bmatrix} l_2^2 m_2 + 2 l_1 l_2 m_2 c_2 + l_1^2 (m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix}. \quad (6.60)$$

Any manipulator mass matrix is symmetric and positive definite, and is, therefore, always invertible.

The velocity term, $V(\Theta, \dot{\Theta})$, contains all those terms that have any dependence on joint velocity. Thus, we obtain

$$V(\Theta, \dot{\Theta}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix}. \quad (6.61)$$

A term like $-m_2 l_1 l_2 s_2 \dot{\theta}_2^2$ is caused by a **centrifugal force**, and is recognized as such because it depends on the square of a joint velocity. A term such as $-2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2$ is caused by a **Coriolis force** and will always contain the product of two different joint velocities.

The gravity term, $G(\Theta)$, contains all those terms in which the gravitational constant, g, appears. Therefore, we have

$$G(\Theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}. \quad (6.62)$$

Note that the gravity term depends only on Θ and not on its derivatives.

6.9 LAGRANGIAN FORMULATION OF MANIPULATOR DYNAMICS

The Newton–Euler approach is based on the elementary dynamic formulas (6.29) and (6.30) and on an analysis of forces and moments of constraint acting between the links. As an alternative to the Newton–Euler method, in this section we briefly introduce the **Lagrangian dynamic formulation.** Whereas the Newton–Euler formulation might be said to be a "force balance" approach to dynamics, the Lagrangian formulation is an "energy-based" approach to dynamics. Of course, for the same manipulator, both will give the same equations of motion. Our statement

We start by developing an expression for the kinetic energy of a manipulator. The kinetic energy of the ith link, k_i, can be expressed as

$$k_i = \frac{1}{2} m_i v_{C_i}^T v_{C_i} + \frac{1}{2}\, {}^i\omega_i^T \, {}^{C_i}I_i \, {}^i\omega_i, \tag{6.69}$$

where the first term is kinetic energy due to linear velocity of the link's center of mass and the second term is kinetic energy due to angular velocity of the link.

The total kinetic energy of the manipulator is the sum of the kinetic energy in the individual links—that is,

$$k = \sum_{i=1}^{n} k_i. \tag{6.70}$$

The v_{C_i} and ${}^i\omega_i$ in (6.69) are functions of Θ and $\dot{\Theta}$, so we see that the kinetic energy of a manipulator can be described by a scalar formula as a function of joint position and velocity, $k(\Theta, \dot{\Theta})$. In fact, the kinetic energy of a manipulator is given by

$$k(\Theta, \dot{\Theta}) = \frac{1}{2} \dot{\Theta}^T M(\Theta) \dot{\Theta}, \tag{6.71}$$

where $M(\Theta)$ is the $n \times n$ manipulator mass matrix already introduced in Section 6.8.

The potential energy of the ith link, u_i, can be expressed as

$$u_i = \; m_i \, {}^0g^T \, {}^0P_{C_i}$$

where ${}^0g$ is the 3×1 gravity vector, ${}^0P_{C_i}$ is the vector locating the center of mass of the ith link,

The total potential energy stored in the manipulator is the sum of the potential energy in the individual links—that is,

$$u = \sum_{i=1}^{n} u_i. \tag{6.74}$$

Because the ${}^0P_{C_i}$ in (6.73) are functions of Θ, we see that the potential energy of a manipulator can be described by a scalar formula as a function of joint position, $u(\Theta)$.

The Lagrangian dynamic formulation provides a means of deriving the equations of motion from a scalar function called the **Lagrangian**, which is defined as the difference between the kinetic and potential energy of a mechanical system. In our notation, the Lagrangian of a manipulator is

$$\mathcal{L}(\Theta, \dot{\Theta}) = k(\Theta, \dot{\Theta}) - u(\Theta). \tag{6.75}$$

The equations of motion for the manipulator are then given by

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\Theta}} - \frac{\partial \mathcal{L}}{\partial \Theta} = \tau, \tag{6.76}$$

where τ is the $n \times 1$ vector of actuator torques. In the case of a manipulator, this equation becomes

$$\frac{d}{dt}\frac{\partial k}{\partial \dot{\Theta}} - \frac{\partial k}{\partial \Theta} + \frac{\partial u}{\partial \Theta} = \tau, \tag{6.77}$$

where the arguments of $k(\cdot)$ and $u(\cdot)$ have been dropped for brevity.

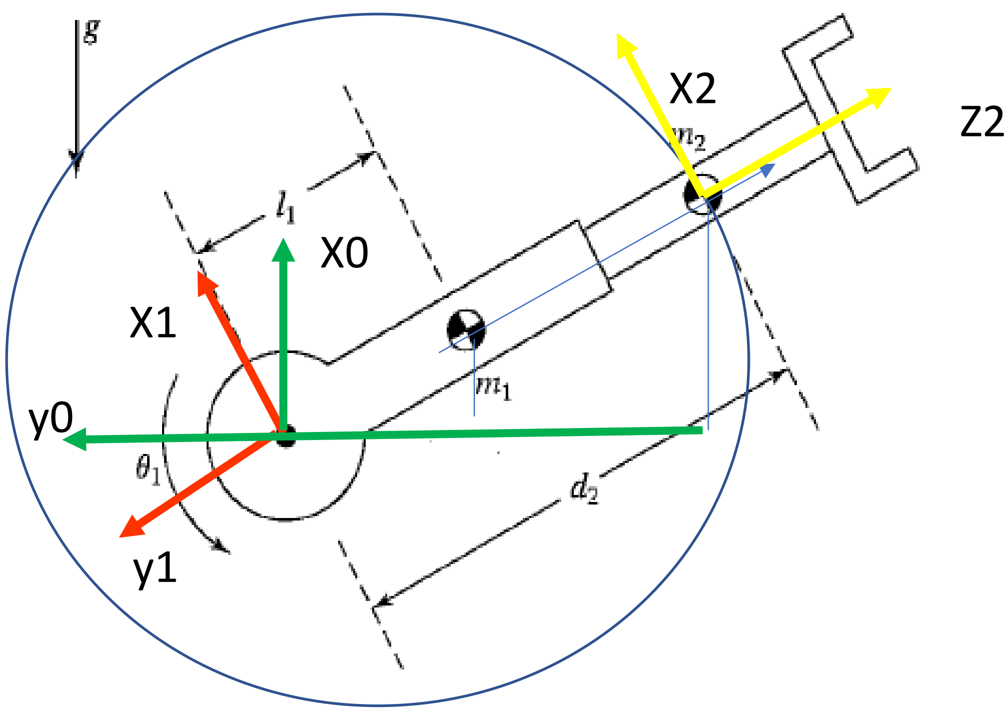
EXAMPLE 6.5

The links of an RP manipulator, shown in Fig. 6.7, have inertia tensors

$$^{C_1}I_1 = \begin{bmatrix} I_{xx1} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix},$$

$$^{C_2}I_2 = \begin{bmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{bmatrix},$$

and total mass m_1 and m_2. As shown in Fig. 6.7, the center of mass of link 1 is located at a distance l_1 from the joint-1 axis, and the center of mass of link 2 is at the variable distance d_2 from the joint-1 axis. Use Lagrangian dynamics to determine the equation of motion for this manipulator.



we write the kinetic energy of link 1 as

$$k_1 = \frac{1}{2}m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} I_{zz1}\, \dot{\theta}_1^2$$

and the kinetic energy of link 2 as

$$k_2 = \frac{1}{2}m_2(d_2^2\dot{\theta}_1^2 + \dot{d}_2^2) + \frac{1}{2} I_{zz2}\, \dot{\theta}_1^2.$$

Hence, the total kinetic energy is given by

$$k(\Theta, \dot{\Theta}) = \frac{1}{2}(m_1 l_1^2 + I_{zz1} + I_{zz2} + m_2 d_2^2)\dot{\theta}_1^2 + \frac{1}{2}m_2 \dot{d}_2^2.$$

we write the potential energy of link 1 as

$$u_1 = m_1 l_1 g \sin(\theta_1)$$

and the potential energy of link 2 as

$$u_2 = m_2 g d_2 \sin(\theta_1)$$

Hence, the total potential energy

$$u(\Theta) = g(m_1 l_1 + m_2 d_2)\sin(\theta_1)$$

Next, we take partial derivatives as needed for (6.77):

$$\frac{\partial k}{\partial \dot{\Theta}} = \begin{bmatrix} (m_1 l_1^2 + I_{zz1} + I_{zz2} + m_2 d_2^2)\dot{\theta}_1 \\ m_2 \dot{d}_2 \end{bmatrix},$$

$$\frac{\partial k}{\partial \Theta} = \begin{bmatrix} 0 \\ m_2 d_2 \dot{\theta}_1^2 \end{bmatrix},$$

$$\frac{\partial u}{\partial \Theta} = \begin{bmatrix} g(m_1 l_1 + m_2 d_2)\cos(\theta_1) \\ g m_2 \sin(\theta_1) \end{bmatrix}.$$

Finally, substituting into (6.77), we have

$$\begin{aligned} \tau_1 &= (m_1 l_1^2 + I_{zz1} + I_{zz2} + m_2 d_2^2)\ddot{\theta}_1 + 2 m_2 d_2 \dot{\theta}_1 \dot{d}_2 \\ &\quad + (m_1 l_1 + m_2 d_2) g \cos(\theta_1), \\ \tau_2 &= m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + m_2 g \sin(\theta_1). \end{aligned}$$

From (6.89), we can see that

$$M(\Theta) = \begin{bmatrix} (m_1 l_1^2 + I_{zz1} + I_{zz2} + m_2 d_2^2) & 0 \\ 0 & m_2 \end{bmatrix},$$

$$V(\Theta, \dot{\Theta}) = \begin{bmatrix} 2 m_2 d_2 \dot{\theta}_1 \dot{d}_2 \\ -m_2 d_2 \dot{\theta}_1^2 \end{bmatrix},$$

$$G(\Theta) = \begin{bmatrix} (m_1 l_1 + m_2 d_2) g \cos(\theta_1) \\ m_2 g \sin(\theta_1) \end{bmatrix}.$$